MATH 3341 — Spring 2021

Lab 09: Ill-conditioned Matrices and Finite Precision Arithmetic

If you haven't downloaded and unzipped `Math.3341.zip`. Download and unzip it under `H:` (H Drive if you are working on the Remote Lab). Change the current working directory by typing `cd H:\Math.3341\Math.3341.Lab.09` in the Command Window, and type `edit lab_09_script` in the Command Window to edit `lab_09_script.m`.

1 Ill-Conditioned Systems

(a) Open the function file `lab_09_function.m`.

(b) Generate an $n \times n$ Hilbert matrix `A`.

(c) Create an $n \times 1$ all-one vector `b`.

(d) Now solve the ill-conditioned linear system $Ax = b$ for x by means of $x = A^{-1}b$:

- Use `invhilb` to find A^{-1}, and store the exact solution x to `x_exact`.
- Use \ to solve the linear system and store the approximate solution $\hat{x}$ to `x_backslash`.
- Use `inv` to find A^{-1}, and store the approximate solution $\hat{x}$ to `x_inv`.

(e) Calculate the infinity norm of relative error of `x_backslash` and `x_inv` using the formula below, and store each relative error to `error_backslash` and `error_inv`, respectively.

$$\text{error}_{\text{relative}} = \frac{\|\hat{x} - x\|_\infty}{\|x\|_\infty}.$$

(f) Calculate the 1-norm condition number of matrix `A` using `cond` and store the result to `cond_A`.

(g) Next, go to `lab_09_script.m`, create a vector named `n`: `n = [9, 11, 13, 15]`. Using a for-loop to call `lab_09_function` by passing each entry of `n` as the input argument. What do you notice about the results?

2 Finite Precision Arithmetic

(a) Use either a for-loop or `sum` to calculate

$$s = \sum_{i=1}^{10} 0.1 = 0.1 + 0.1 + \cdots + 0.1.$$

Then calculate the error `error_1 = abs(s - 1)`.

(b) Calculate

$$b = 2 - 3\left(\frac{4}{3} - 1\right).$$

Then calculate the error `error_2 = abs(b - 1)`.

(c) Calculate each side of the following equation in the variables `lhs` and `rhs`:

$$1 + a + a^2 + a^3 + a^4 + a^5 = \frac{1 - a^6}{1 - a},$$

for the value $a = 0.3$. Then calculate the error `error_3 = abs(lhs - rhs)`. Hint: you can use either a for-loop or `polyval` to calculate `lhs`.

(d) Calculate the following and compare their output.

```
x = 1e16 + 1 - 1e16;
y = 1e16 - 1e16 + 1;
z = 1e16 - (1e16 - 1);
```

(e) Calculate the following and compare their output.

```
u = 1 + 0.1 - 1;
v = 1 - 1 + 0.1;
w = 1 - (1 - 0.1);
```

Once you finish, call `diary('lab_09_output.txt')`, then run the script file `lab_09_script.m`, and call `diary off` to save the resulting output. Then you need to upload files `lab_09_script.m`, `lab_09_function.m`, and `lab_09_output.txt` to Overleaf, and answer the corresponding questions in `body.tex`.