MATH 3341: Introduction to Scientific Computing Lab

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May 06, 2020

Lab 13: Random Numbers, Histogram and Monte Carlo Integration

Random Numbers and Histogram

`rand`: Uniformly distributed pseudorandom numbers.

- `R = rand(N)` returns an N-by-N matrix containing pseudorandom values drawn from the standard uniform distribution on the open interval(0,1).
- `rand(M,N)` or `rand([M,N])` returns an M-by-N matrix.
- `rand(M,N,P,...)` or `rand([M,N,P,...])` returns an M-by-N-by-P-by-... array.
- `rand` returns a scalar.
- `rand(SIZE(A))` returns an array the same size as A.

`randn`: Normally distributed pseudorandom numbers.

- `R = randn(N)` returns an N-by-N matrix containing pseudorandom values drawn from the standard normal distribution.
- `randn(M,N)` or `randn([M,N])` returns an M-by-N matrix.
- `randn(M,N,P,...)` or `randn([M,N,P,...])` returns an M-by-N-by-P-by-... array. randn returns a scalar.
- `randn(SIZE(A))` returns an array the same size as A.

`randi`: Pseudorandom integers from a uniform discrete distribution.

- `R = randi(IMAX,N)` returns an N-by-N matrix containing pseudorandom integer values drawn from the discrete uniform distribution on 1:IMAX.
- `randi(IMAX,M,N)` or `randi(IMAX,[M,N])` returns an M-by-N matrix.
- `randi(IMAX,M,N,P,...)` or `randi(IMAX,[M,N,P,...])` returns an M-by-N-by-P-by-... array.
- `randi(IMAX)` returns a scalar.
- `randi(IMAX,SIZE(A))` returns an array the same size as A.

`histogram`: Plots a histogram.

- `histogram(X)` plots a histogram of X. histogram determines the bin edges using an automatic binning algorithm that returns uniform bins of a width that is chosen to cover the range of values in `X` and reveal the shape of the underlying distribution.
- `histogram(X,M)`, where `M` is a scalar, uses `M` bins.
- `histogram(X,EDGES)`, where `EDGES` is a vector, specifies the edges of the bins.

Monte Carlo Integration

1-D Monte Carlo Integration

$$\begin{aligned}\int_a^b f(x)\,dx &= \int_a^b \frac{f(x)}{p(x)}p(x)\,dx \\ &= E[f(X)/p(X)] \\ &\approx \frac{1}{N}\sum_{i=1}^{N}\frac{f(x_i)}{p(x_i)} \\ &= \frac{1}{N}\sum_{i=1}^{N}\frac{f(x_i)}{1/(b-a)} \\ &= \frac{b-a}{N}\sum_{i=1}^{N} f(x_i),\end{aligned}$$

where $x_1, x_2, \ldots, x_N$ are uniformly distributed on $[a, b]$, hence $p(x_i) = \frac{1}{b-a}, i = 1, 2, \ldots, N.$

2-D Monte Carlo Integration

$$
\begin{aligned}
\int_a^b \int_c^d f(x,y)\,dy\,dx &= \int_a^b \int_c^d \frac{f(x,y)}{p(x,y)} p(x,y)\,dy\,dx \\
&= \int_a^b \int_c^d \frac{f(x,y)}{p_X(x)p_Y(y)} p_X(x)p_Y(y)\,dy\,dx \\
&= \int_a^b \frac{1}{p_X(x)} \int_c^d \frac{f(x,y)}{p_Y(y)} p_Y(y)\,dy\,p_X(x)\,dx \\
&= \int_a^b \frac{1}{p_X(x)} E[f(x,Y)/p_Y(Y)] p_X(x)\,dx \\
&\approx \int_a^b \frac{1}{p_X(x)} \frac{1}{N} \sum_{j=1}^{N} \frac{f(x,y_j)}{p_Y(y_j)} p_X(x)\,dx \\
&\approx \frac{1}{M} \sum_{i=1}^{M} \frac{1}{p_X(x_i)} \frac{1}{N} \sum_{j=1}^{N} \frac{f(x_i,y_j)}{p_Y(y_j)}
\end{aligned}
$$

2-D Monte Carlo Integration

$$\begin{aligned}\int_a^b \int_c^d f(x,y)\,dy\,dx &\approx \frac{1}{M}\sum_{i=1}^{M}\frac{1}{p_X(x_i)}\frac{1}{N}\sum_{j=1}^{N}\frac{f(x_i,y_j)}{p_Y(y_j)}\\ &= \frac{1}{MN}\sum_{i=1}^{M}\frac{1}{1/(b-a)}\sum_{j=1}^{N}\frac{f(x_i,y_j)}{1/(d-c)}\\ &= \frac{(b-a)(d-c)}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}f(x_i,y_j),\end{aligned}$$

where X and Y are independent and identically uniformly distributed, hence $p(x,y) = p_X(x)p_Y(y)$, and $p_X(x) = \frac{1}{b-a}$, $p_Y(y) = \frac{1}{d-c}$.