MATH 3340 - Scientific Computing Homework 2

Due: Friday, 02/21/2020, 02:00 PM

Please note that the deadline will be enforced as per the first homework. Remember that you are allowed to work in teams of two on this assignment. You are encouraged to prepare your work in LaTeX; a template will be provided to help you put it all together. If you choose to submit a hard copy, you may submit only one copy for a team, indicating the names of both contributors. Online submission is encouraged, however, in that case both members of a team should submit the PDF file containing their work and showing both their names.

Instruction

1. Go to `https://www.overleaf.com` and sign in (required).
2. Click *Menu* (up left corner), then *Copy Project*.
3. Go to `LaTeX/meta.tex` (the file `meta.tex` under the folder `LaTeX`) to change the section and your name, e.g.,
   - change title to `\title{MATH 3340-01 Scientific Computing Homework 2}`
   - change author to `\author{Albert Einstein \& Carl F. Gauss}`
4. For Problem 1, 2, 3, 4, you need to write fuction/script files and store results to output files. Here are suggested names for function files, script files, and output files:

Problem	Function File	Script File	Output File
1	`bisection.m`	`hw2_p1.m`	`hw2_p1.txt`
2	`newton.m`	`hw2_p2.m`	`hw2_p2.txt`
3 (a)	`bisectionImproved.m`	`hw2_p3_a.m`	`hw2_p3_a.txt`
3 (b)	`newton.m`	`hw2_p3_b.m`	`hw2_p3_b.txt`
4	`bisectionRecursive.m`	`hw2_p4.m`	`hw2_p4.txt`

   Once finished, you need to upload these files to the folder `src` on Overleaf. If you have different filenames, please update the filenames in `\lstinputlisting{../src/your_script_name.m}` accordingly. You can use the MATLAB script `src/save_output.m` to generate the output files automatically (the script filenames should be exactly same as listed above).
5. For Problem 3(a), 3(b), 5, you can either print the recompiled PDF out and write your solution/explanation there or directly type in the Overleaf and print it out at the end.
6. Recompile, download, and print out the generated PDF.
7. You may find LaTeX.Mathematical.Symbols.pdf and the second part of Lab 01 Slides and Lab 02 Slides helpful.

Problem 1. Create a MATLAB function that uses the bisection method to find the root of $f(x)$. The function header for the bisection method could read:

```
function [r, iters] = bisection(f, xL, xR)
```

where the inputs are:

```
f  = the function f(x) for which you want to find the root
xL = left limit of the interval
xR = right limit of the interval
```

and the outputs are:

```
r     = the root
iters = number of iterations performed
```

This is a *minimal* list of inputs and outputs required for your function. Additional inputs or outputs may be necessary and they are left to your discretion. Use this code to find the root of the piecewise function:

$$f(x) = \begin{cases} x^3 + 3x + 1 & \text{if } x \leq 0, \\ 1 + \sin(x) & \text{if } x > 0, \end{cases}$$

on the interval $x_L = -2$, $x_R = 0.1$. For this problem submit

- Your function file defining the bisection method
- The script file that calls this function
- A text file that contains the following output:
  - the value of the root x^*
  - the value of $f(x^*)$
  - the number of iterations performed

  The output must clearly identify each one of these results.

Note: Your script file should define the inputs of your function, call your function, and contain any additional formatting needed for pretty-printing the output.

Problem 2. Using the code in Problem 1 as a template, write a different MATLAB function which now implements Newton's Method to find the root of a function. Use this new code to find the root of

$$f(x) = x^3 + 3x + 1$$

with an initial guess $x_0 = -2$ and an accuracy $\alpha = 10^{-5}$. For this problem submit the equivalent files as outlined in Problem 1: a function file, script file, and output file. Your output must also meet the same requirements: it must list the value of the root, function value at the root, and number of iterations performed.

Problem 3. Now use both codes you developed above (Newton and bisection) to find the roots of

$$f(x) = \frac{1}{2} + \frac{x^2}{4} - x\sin(x) - \frac{\cos(2x)}{2}.$$

(a) **Using bisection method:** on the interval $x_L = 0$, $x_R = \pi$. Do you encounter any problems? Is there a way your code can be improved to behave appropriately for this problem? Explain any changes you choose to make.

(b) **Using Newton's method:** with the following initial guesses:

- $x_0 = \pi/2$,
- $x_0 = 5\pi$,
- $x_0 = 10\pi$.

Use an accuracy of $\alpha = 10^{-5}$, and a maximum number of 20000 iterations. Try to explain the different behavior for the three starting values.

For each part, submit the equivalent files as outlined in Problem 1: a function file, script file, and output file. Your output must again align to the same requirements.

Problem 4. Write a recursive MATLAB function that implements the bisection method to find the root of a given function $f(x)$. The function header for the bisection method should be similar to the one in the first problem, using the same inputs as a minimum and producing the same outputs. You may find it necessary to use some extra variable or variables as input arguments. Run this code on the function:

$$f(x) = x^2 - 7$$

with the initial bounds $x_L = -1$, $x_R = 9$. For this problem submit again files corresponding to those outlined Problem 1: a function file, script file, and output file (same quantities should be printed out in the output).

Problem 5. This is a problem to be solved only analytically. Think about Newton's method and its disadvantage: it requires the expression for the derivative of the function. Develop a method that circumvents this problem by using an approximation to the derivative obtained using a secant to the graph of the function: suppose you start with two points x_0 and x_1. Your approximation to the root, x_2, will be the intersection of the secant through $(x_0, f(x_0))$ and $(x_1, f(x_1))$ with the x-axis. Express x_2 as a function of x_0 and x_1, then show how this can be generalized from $\{x_0, x_1, x_2\}$ to $\{x_{k-1}, x_k, x_{k+1}\}$.