MATH 3340 - Scientific Computing Homework 1

Due: Wednesday, 02/12/2020, 02:00 PM

Problem 1. Do Ex. 1, Ex. 2 and Ex. 3 on page 16 in the lecture notes. The calculations in these exercises are to be done without the use of MATLAB (although you may use MATLAB to check your answers). Print all your work cleanly and in an organized fashion. This should include explanation at key steps (elaborate in your own words, as appropriate) and general forms of any formulae you're using.

(a) (Ex. 1) Compute the dot product $\mathbf{x}^T\mathbf{y}$ and the inner products $(\mathbf{x}, \mathbf{y})$ and $(\mathbf{x}, \mathbf{x})$ for the two vectors defined by:

$$\mathbf{x} = \begin{bmatrix} 1-i \\ 2 \\ 3+2i \\ 4i \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} 2i \\ -1 \\ 2-3i \\ 1+i \end{bmatrix}.$$

(b) (Ex. 2) Find the eigenvalues of the diagonal matrix Dj given by:

$$D = \begin{bmatrix} 2 & 0 \\ 0 & -5 \end{bmatrix}.$$

Generalize to the case of a diagonal matrix of order n.

(c) (Ex. 3) Calculate $A\mathbf{x}$ and AB for the quantities given below. Is the product BA equal to AB?

$$A = \begin{bmatrix} 2 & 1 & 2 \\ -1 & 1 & 3 \\ -2 & 3 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 7 & 2 \\ 3 & 3 & 5 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}.$$

Then compute the *outer product* X of $\mathbf{x}$ with itself; it is defined by $X = \mathbf{x}\mathbf{x}^T$.

Problem 2. Calculate the product $C = AB$ of the two matrices A and B given below. For each entry in the product matrix show how all the pertinent calculations, i.e. write out clearly the sums and products involved.

$$A = \begin{bmatrix} 2 & -3 & 1 & 7 \\ -1 & 3 & 5 & -2 \\ 1 & -1 & 4 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 2 \\ 1 & -1 \\ 4 & 1 \\ -2 & 5 \end{bmatrix}.$$

Problem 3. Do Ex. 1 through Ex. 4 on page 39 in the lecture notes. For each of these problems you can print out the code snippet that performs the task; use a separate script each time.

(a) (Ex. 1) Create an anonymous MATLAB function that computes the value of $f(x)$, defined piecewise as:

$$f(x) = \begin{cases} x+1 & x \leq 0, \\ \sin(x) & x > 0. \end{cases}$$

(b) (Ex. 2) Write a MATLAB code that can be used to compute an approximation to e^2 using the first five terms of the Taylor series for the exponential around the point $x = 0$, that is:

$$e^x \approx 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}.$$

(c) (Ex. 3) Use the MATLAB help system to learn about the function `isprime`, then use this function together with `find` and the colon operator to produce a list of all the prime numbers less than or equal to $M = 30$.

(d) (Ex. 4) To investigate the roots of an equation of the form $f(x) = 0$, it may be helpful to plot the function $f(x)$ in an interval of interest. Use this method to find out if the following three functions have any root in the interval $[0, 2]$: $f_1(x) = e^x + x^2 - 5$, $f_2(x) = x^3 - 5x^2 + 2$ and $f_3(x) = \ln(x^2 + 2) - x^2$.

Problem 4. Convert to base two the following integers and floating-point numbers: 5, 13, 5.25 and 13.75. Do these calculations by hand, and explain your answers.

Problem 5. Read about integer and floating-point number representation on digital computers. You'll find information on the internet as well as printouts in the class materials. Then answer the following three questions:

(a) what is the range for an unsigned integer represented on 64 bits?
(b) what is the range of a signed integer that is represented on 64 bits?
(c) what is the range of a double precision floating-point number represented using the IEEE-754 standard with a mantissa of 52 digits?