

MATH 3341: Introduction to Scientific Computing Lab

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Lab 08: MATLAB Interpolation Routines &
Their Derivatives



The background features a large, light gray watermark of the University of Wyoming seal. The seal is circular with a rope-like border. Inside the border, the words "UNIVERSITY OF WYOMING" are written in an arc at the top, and "1886" is at the bottom. In the center, there is an open book with a banner across it that says "EQUALITY".

Polynomial Interpolation Routines



polyfit and polyval

- $p = \text{polyfit}(\text{xdata}, \text{ydata}, n)$: finds the coefficients of a polynomial $p(x)$ of degree n , i.e., $p(x) = p_1x^n + p_2x^{n-1} + \dots + p_nx + p_{n+1}$, that fits the data xdata , ydata best in a least-squares sense. p is a row vector of length $n + 1$ containing the polynomial coefficients in descending powers, p stores $[p_1, p_2, \dots, p_n, p_{n+1}]$.
- $y = \text{polyval}(p, x)$: returns the value of a polynomial p evaluated at x : $y = p(x) = p_1x^n + p_2x^{n-1} + \dots + p_nx + p_{n+1}$.
- Example:

```
xdata = [-2, 0, 1]
```

```
ydata = [9, 1, 3]
```

```
p = polyfit(xdata, ydata, 2) % p = [2, 0, 1]
```

```
y = polyval(p, 2) % y = 9
```

In other words, the fitted polynomial is

$p(x) = 2x^2 + 0x + 1 = 2x^2 + 1$, and evaluate $p(x)$ at $x = 2$, we have $y = p(2) = 2 \times 2^2 + 1 = 9$.



Piecewise Polynomial: `spline`, `pchip`, and `ppval`

- `pp = spline(xdata, ydata)`: Use cubic spline (piecewise cubic polynomial) to fit the data `xdata` and `ydata`. `pp` is a struct (structure) contains number of pieces of cubic polynomials (`pp.pieces`), coefficients matrix (`pp.coefs`) of which the `i`th row are the coefficients for the `i`th piece cubic polynomial, break points (`pp.breaks`) which is a row vector contains the endpoints of the interval for each pieces.
- `pp = pchip(xdata, ydata)`: Use Piecewise Cubic Hermite Interpolating Polynomial to fit the data `xdata` and `ydata`. `pp` is same as above.
- `y = ppval(pp, x)`: determines which intervals `x` lies on and then evaluate the corresponding cubic polynomial at `x`.
- `y = spline(xdata, ydata, x)`: is the same as `y = ppval(spline(xdata, ydata), x)`, thus providing, in `y`, the values of the interpolant at `x`.



Piecewise Polynomial: `spline`, `pchip`, and `ppval`

Example:

```
xdata = [0 1 2 3]
ydata = [10 8 6 4]
pp = spline(xdata, ydata)
y = ppval(pp, 1.5)           % y = 7
y = spline(xdata, ydata, 1.5) % same as y = ppval(pp, 1.5)
```





Derivatives of Interpolation Polynomials



polyder: Differentiate polynomial

- `dp = polyder(p)`: returns the derivative of the polynomial whose coefficients are the elements of vector `p`.
- Example:

```
p = [4 3 2 1]
```

```
dp = polyder(p) % dp = [12 6 2]
```

That is, given a polynomial $p(x) = 5x^3 + 3x^2 + 2x + 1$, the derivative with respect to x is $p'(x) = dp(x) = 12x^2 + 6x + 2$.

