Full Name: ______________________________ **W-Number:** ____________________

MATH 2205: Calculus II – Sample Exam 3

Summer 2019 – Friday, July 05, 2019

Instructions:

- Show all your work and use the space provided on the exam. Correct mathematical notation is required and all partial credit is at discretion of the grader.
- Write neatly and make sure your work is organized.
- Make certain that you have written your Full Name and W-Number in the spaces provided at the top of the exam. Failure to do so may result in a loss of points.
- No aids beyond a scientific, non-graphing calculator are allowed. This means no notes, no cell phones, etc., are permitted during the exam.
- Present your Photo I.D. when turning in your exam.
- The exam has 11 pages. Please check to see that your copy has all the pages.

For Instructor Use Only

Question	1	2	3	4	5	6	7	8	9	Total
Points	15	20	10	10	10	10	10	15	10	100
Mark										

Formulae you may find useful:

- $\displaystyle\sum_{k=1}^{n} c = cn, \quad \sum_{k=1}^{n} k = \frac{n(n+1)}{2}, \quad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}.$
- $\displaystyle\int x^p \, dx = \frac{x^{p+1}}{p+1} + C$, where $p \neq -1$.
- $\displaystyle\int x^{-1} \, dx = \ln|x| + C.$
- $\displaystyle\int e^x \, dx = e^x + C.$
- $\displaystyle\int \sin x \, dx = -\cos x + C.$
- $\displaystyle\int \cos x \, dx = \sin x + C.$
- $\displaystyle\int \frac{1}{1+x^2} \, dx = \arctan x + C.$
- $\displaystyle\int \frac{1}{\sqrt{1-x^2}} \, dx = \arcsin x + C.$
- $\displaystyle\int \sec x \tan x \, dx = \sec x + C.$
- $\displaystyle\int \sec^2 x \, dx = \tan x + C.$
- $\displaystyle\cos^2\theta = \frac{1+\cos 2\theta}{2}.$
- $\displaystyle\sin^2\theta = \frac{1-\cos 2\theta}{2}.$
- $\sin^2\theta + \cos^2\theta = 1.$
- $\sin 2\theta = 2\sin\theta\cos\theta.$
- $\cos 2\theta = \cos^2\theta - \sin^2\theta.$
- Midpoint Rule: $\displaystyle M(n) = \sum_{k=1}^{n} f\left(\frac{x_{k-1}+x_k}{2}\right)\Delta x.$
- Trapezoid Rule: $\displaystyle T(n) = \left[\frac{1}{2}f(x_0) + \sum_{k=1}^{n-1} f(x_k) + \frac{1}{2}f(x_n)\right]\Delta x.$
- Simpson's Rule: $\displaystyle S(n) = \sum_{k=0}^{n/2-1} [f(x_{2k}) + 4f(x_{2k+1}) + f(x_{2k+2})]\frac{\Delta x}{3}.$

1. (15 points) Circle TRUE if the statement is true or FALSE if it is not, and justify your choice briefly.

   (a) TRUE or FALSE: Integration by Parts can be used to write $\int x^2e^{-x}\,dx = -x^2e^{-x} + \int 2xe^{-x}\,dx$.
   Explanation:

   (b) TRUE or FALSE: If the interval of convergence of the series $\sum_{k=1}^{\infty} c_k x^k$ is $(-3,3)$, then the interval of convergence of $\int \left(\sum_{k=1}^{\infty} c_k x^k\right) dx$ is also $(-3,3)$.
   Explanation:

   (c) TRUE or FALSE: The Fundamental Theorem of Calculus uses the derivative of the function f to evaluate the definite integral $\int_a^b f(x)\,dx$.
   Explanation:

   (d) TRUE or FALSE: If $\sum_{k=0}^{\infty}|a_k|$ converges, then $\sum_{k=0}^{\infty}(\sin k \cdot a_k)$ must converge.
   Explanation:

   (e) TRUE or FALSE: $\dfrac{[3(k+1)]!}{(3k)!} = 3k+1$.
   Explanation:

2. (20 points) Evaluate the following sums.

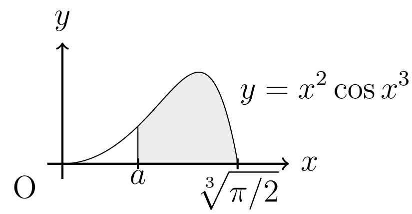
(a) $\displaystyle\sum_{k=10}^{100}(k+5)(k+4)$.

(b) $\displaystyle\sum_{k=0}^{\infty}\frac{5}{(5k+1)(5k+6)}$.

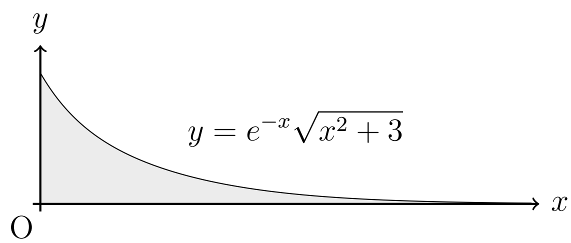
(c) $\dfrac{3}{7} + \dfrac{6}{21} + \dfrac{12}{63} + \dfrac{24}{189} + \cdots$.

(d) $\displaystyle\sum_{k=0}^{\infty} \left(e^{-2k+1} + \frac{1}{\pi^{3k-1}} \right)$.

3. (10 points) Determine the value of the positive parameter a so that the area of the shaded region in the picture is equal to $\frac{1}{3}$.

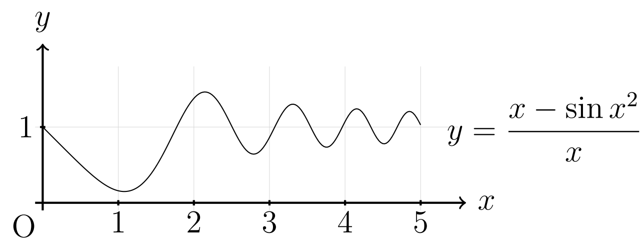


4. (10 points) Consider the infinitely long shaded region R indicated in the picture. Determine the volume of the solid of the revolution obtained when R is revolved about the x-axis.



5. (10 points) Find the interval of convergence and radius of convergence for the power series given by
$$\sum_{k=1}^{\infty} \frac{(-1)^k (x-3)^k}{k^{2/3}}.$$

6. (10 points) Use MacLaurin Series to approximate the net area between the function $y = \dfrac{x - \sin x^2}{x}$ and the x-axis from $x = 0$ to $x = 2$ with an error no greater than $10^{-4} = 0.0001$. Be sure to justify that your error satisfies the given bound.



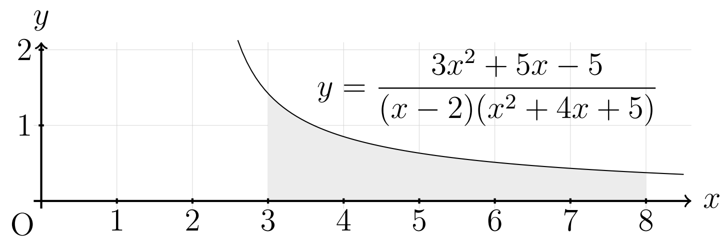
7. (10 points) Find power series representations centered at 0 for $\ln\left(\dfrac{1+8x^3}{1-x^3}\right)$ and give its interval of convergence.

8. (15 points) Determine whether the following series converge.

(a) $\displaystyle\sum_{k=1}^{\infty}\left(\frac{4k^3+1000k-3}{9k^3+20k^2+6}\right)^k$.

(b) $\displaystyle\sum_{k=1}^{\infty}\frac{(2k)^{2k}}{(2k)!}$.

9. (Bonus, 10 points) Compute the area of region R bounded by $y = \dfrac{3x^2+5x-5}{(x-2)(x^2+4x+5)}$, x-axis, $x=3$, $x=8$.



Topics for Midterm Exam 1:

- Sigma Notation;
- Left/midpoint/right Riemann Sum;
- Definite Integral;
- Fundamental Theorem of Calculus;
- Integrating with Even and Odd Functions;
- Mean Value Theorem for Integrals;
- Substitution Rule for Indefinite/Definite Integrals;
- Area of the Region Between Curves (Integrating with respect to x or y);
- Volume of the Solid by Slicing (Disk/Washer Method) and by Shell Method;
- Length of Curves;
- Surface Area;
- Physical Applications: Mass of a One-Dimensional Object, Work (Hooke's law).

Topics for Midterm Exam 2:

- Integration by Parts: Integrate product of x^p and $(\ln x)^q$, product of x^p and e^{ax}, $\sin^n x$ or $\cos^n x$, product of e^{ax} and $\sin^n x$ or $\cos^n x$;
- Trigonometric Integrals: Integrate $\sin^n x$, $\cos^n x$, and $\sin^m x \cos^n x$;
- Trigonometric Substitutions: Integrals involving $a^2 - x^2$;
- Partial Fractions with Simple/Repeated Linear Factors, and Simple/Repeated Irreducible Quadratic Factors;
- Numerical Integration (bonus problem): Absolute/Relative Error, Midpoint Rule approximation, Trapezoid Rule approximation, Simpson's Rule approximation;
- Improper Integrals: Improper Integrals over Infinite Intervals, Improper Integrals with Unbounded Integrand;
- Differential Equations: Verifying Solutions, Order of a Differential Equation, First- and Second-Order Linear Differential Equations, Initial Value Problems, Solution of a First-Order Linear Differential Equation, Separable First-Order Differential Equations;
- Sequences: Explicit Formulas, Recurrence Relations, Limit of a Sequence;
- Infinite Series: Partial Sums, Geometric Series.

Topics for Final Exam (also include Topics for Midterm Exam 1 & 2):

- Infinite Series: Partial Sums, Geometric Series, Harmonic Series, Alternating Harmonic Series, and p-Series, Telescoping Series;
- Divergence, Integral, Ratio, Root, Comparison, Limit Comparison, Alternating Series Tests;
- Properties of Convergent Series, Absolute and Conditional Convergence;
- Power Series: Interval of Convergence, Radius of Convergence;
- Combining/Differentiating/Integrating Power Series;
- Taylor Polynomial, Taylor/MacLaurin Series;
- Remainder in Alternating Series.