Full Name: ______________________________ **W-Number:** ____________________

MATH 2205: Calculus II – Sample Exam 2

Summer 2019 – Friday, June 21, 2019

Instructions:

- Show all your work and use the space provided on the exam. Correct mathematical notation is required and all partial credit is at discretion of the grader.
- Write neatly and make sure your work is organized.
- Make certain that you have written your Full Name and W-Number in the spaces provided at the top of the exam. Failure to do so may result in a loss of points.
- No aids beyond a scientific, non-graphing calculator are allowed. This means no notes, no cell phones, etc., are permitted during the exam.
- Present your Photo I.D. when turning in your exam.
- The exam has 11 pages. Please check to see that your copy has all the pages.

For Instructor Use Only

Question	1	2	3	4	5	6	7	8	9	Total
Points	15	20	10	10	10	10	10	15	10	100
Mark										

Formulae you may find useful:

- $\sum_{k=1}^{n} c = cn, \quad \sum_{k=1}^{n} k = \frac{n(n+1)}{2}, \quad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}.$
- $\int x^p \, dx = \frac{x^{p+1}}{p+1} + C$, where $p \neq -1$.
- $\int x^{-1} \, dx = \ln|x| + C.$
- $\int e^x \, dx = e^x + C.$
- $\int \sin x \, dx = -\cos x + C.$
- $\int \cos x \, dx = \sin x + C.$
- $\int \frac{1}{1+x^2} \, dx = \arctan x + C.$
- $\int \frac{1}{\sqrt{1-x^2}} \, dx = \arcsin x + C.$
- $\int \sec x \tan x \, dx = \sec x + C.$
- $\int \sec^2 x \, dx = \tan x + C.$
- $\cos^2 \theta = \frac{1+\cos 2\theta}{2}.$
- $\sin^2 \theta = \frac{1-\cos 2\theta}{2}.$
- $\sin^2 \theta + \cos^2 \theta = 1.$
- $\sin 2\theta = 2 \sin \theta \cos \theta.$
- $\cos 2\theta = \cos^2 \theta - \sin^2 \theta.$
- Midpoint Rule: $M(n) = \sum_{k=1}^{n} f\left(\frac{x_{k-1}+x_k}{2}\right) \Delta x.$
- Trapezoid Rule: $T(n) = \left[\frac{1}{2} f(x_0) + \sum_{k=1}^{n-1} f(x_k) + \frac{1}{2} f(x_n)\right] \Delta x.$
- Simpson's Rule: $S(n) = \sum_{k=0}^{n/2-1} \left[f(x_{2k}) + 4f(x_{2k+1}) + f(x_{2k+2})\right] \frac{\Delta x}{3}.$

1. (15 points) Circle TRUE if the statement is true or FALSE if it is not, and justify your choice briefly.

   (a) TRUE or FALSE: The partial fraction decomposition of $\dfrac{2x^3 - 4x^2 + x + 3}{(x^2+1)(x-2)^2}$ is $\dfrac{1}{x-2} + \dfrac{x+1}{x^2+1}$.

   (b) TRUE or FALSE: Use integration by parts, one can show that $\int xg'(x)\,dx = xg(x) - \int g(x)\,dx$.

   (c) TRUE or FALSE: The function $y = e^{2t} - e^{-2t}$ is a solution of the differential equaiton $y'' - 4y = 0$.

   (d) TRUE or FALSE: The differential equation $y''(x) + (\ln x \cdot e^{e^x})y'(x) = y(x) + \cos x \sin e^x$ is NOT linear.

   (e) TRUE or FALSE: If $a_k > 0$ and $\sum_{k=1}^{\infty} a_k$ converges, then $\sum_{k=1}^{\infty}(-1)a_k$ diverges.

2. (20 points) Evaluate the following integrals and sums.

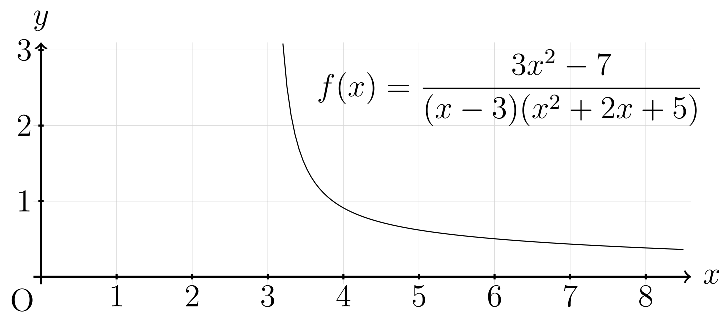
   (a) $\displaystyle\int x^2(\ln x)^2\,dx.$

   (b) $\displaystyle\int_0^{\pi/2} \sin^3 x \cos^4 x\,dx.$

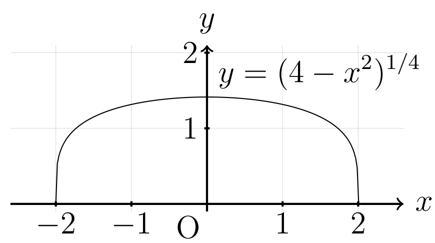
(c) $\displaystyle\sum_{k=1}^{\infty}\left(-\frac{5}{3}\right)^{-k}$.

(d) $\displaystyle\frac{1}{16}+\frac{3}{64}+\frac{9}{256}+\frac{27}{1024}+\cdots$.

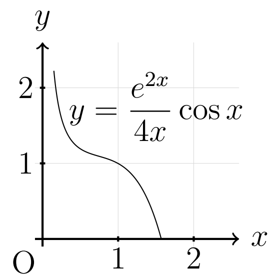
3. (10 points) Compute the area of region R bounded by $f(x) = \dfrac{3x^2 - 7}{(x-3)(x^2+2x+5)}$, x-axis, $x = 4$, $x = 8$. Indicate (by shading) the region R in the graph.



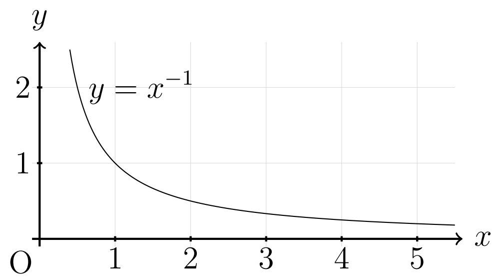
4. (10 points) Let R be the region bounded by $y = (4 - x^2)^{1/4}$ and x-axis. Indicate (by shading) the region R in the graph. Determine the volume of the solid of revolution obtained when R is revolved about the x-axis.



5. (10 points) Let R be the region bounded by $y = \dfrac{e^{2x}}{4x}\cos x$ and x-axis on $[1/5, \pi/2]$. Indicate (by shading) the region R in the graph. Determine the volume of the solid of revolution obtained when R is revolved about the y-axis.



6. (10 points) Let R be the region bounded by the graph of $y = x^{-1}$ and the x-axis, for $x \geq 1$. Indicate (by shading) the region R in the graph. What is the surface area of the solid generated when R is revolved about the x-axis?



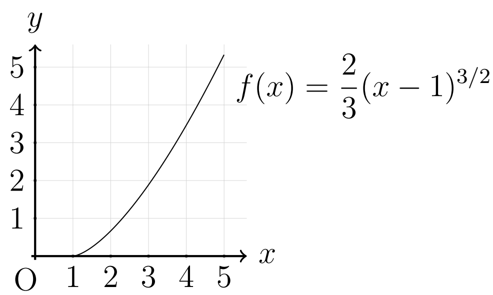
7. (10 points) Write $1.0\overline{95} = 1.095959595\ldots$ as a geometric series and express its value as fraciton.

8. (15 points) A community of hares on an island has a population of 70 when observations begin (at $t = 0$). The population is modeled by the initial value problem shown below.

$$\frac{dP}{dt} = 0.07P\left(1 - \frac{P}{280}\right), \quad P(0) = 70.$$

(a) Find the solution of the initial value problem, for $t \geq 0$.
(b) What is the steady-state population?

9. (Bonus, 10 points) Approximate the arc length of $f(x) = \frac{2}{3}(x-1)^{3/2}$ on the interval $[1, 5]$ using Simpson's Rule with $n = 4$ subintervals.



Topics for Midterm Exam 1:

- Sigma Notation;
- Left/midpoint/right Riemann Sum;
- Definite Integral;
- Fundamental Theorem of Calculus;
- Integrating with Even and Odd Functions;
- Mean Value Theorem for Integrals;
- Substitution Rule for Indefinite/Definite Integrals;
- Area of the Region Between Curves (Integrating with respect to x or y);
- Volume of the Solid by Slicing (Disk/Washer Method) and by Shell Method;
- Length of Curves;
- Surface Area;
- Physical Applications: Mass of a One-Dimensional Object, Work (Hooke's law).

Topics for Midterm Exam 2 (also include Topics for Miterm Exam 1):

- Integration by Parts: Integrate product of x^p and $(\ln x)^q$, product of x^p and e^{ax}, $\sin^n x$ or $\cos^n x$, product of e^{ax} and $\sin^n x$ or $\cos^n x$;
- Trigonometric Integrals: Integrate $\sin^n x$, $\cos^n x$, and $\sin^m x \cos^n x$;
- Trigonometric Substitutions: Integrals involving $a^2 - x^2$;
- Partial Fractions with Simple/Repeated Linear Factors, and Simple/Repeated Irreducible Quadratic Factors;
- Numerical Integration (bonus problem): Absolute/Relative Error, Midpoint Rule approximation, Trapezoid Rule approximation, Simpson's Rule approximation;
- Improper Integrals: Improper Integrals over Infinite Intervals, Improper Integrals with Unbounded Integrand;
- Differential Equations: Verifying Solutions, Order of a Differential Equation, First- and Second-Order Linear Differential Equations, Initial Value Problems, Solution of a First-Order Linear Differential Equation, Separable First-Order Differential Equations;
- Sequences: Explicit Formulas, Recurrence Relations, Limit of a Sequence;
- Infinite Series: Partial Sums, Geometric Series.