Full Name: ____________________________ **W-Number:** ____________________

MATH 2205: Calculus II – Midterm Exam 2

Summer 2019 – Friday, June 21, 2019

Instructions:

- Show all your work and use the space provided on the exam. Correct mathematical notation is required and all partial credit is at discretion of the grader.
- Write neatly and make sure your work is organized.
- Make certain that you have written your Full Name and W-Number in the spaces provided at the top of the exam. Failure to do so may result in a loss of points.
- No aids beyond a scientific, non-graphing calculator are allowed. This means no notes, no cell phones, etc., are permitted during the exam.
- Present your Photo I.D. when turning in your exam.
- The exam has 10 pages. Please check to see that your copy has all the pages.

For Instructor Use Only

Question	1	2	3	4	5	6	7	8	9	Total
Points	15	20	10	10	10	10	10	15	15	100
Mark										

Formulae you may find useful:

- $\sum_{k=1}^{n} c = cn, \quad \sum_{k=1}^{n} k = \frac{n(n+1)}{2}, \quad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}.$
- $\int x^p \, dx = \frac{x^{p+1}}{p+1} + C$, where $p \neq -1$.
- $\int x^{-1} \, dx = \ln|x| + C.$
- $\int e^x \, dx = e^x + C.$
- $\int \sin x \, dx = -\cos x + C.$
- $\int \cos x \, dx = \sin x + C.$
- $\int \frac{1}{1+x^2} \, dx = \arctan x + C.$
- $\int \frac{1}{\sqrt{1-x^2}} \, dx = \arcsin x + C.$
- $\int \sec x \tan x \, dx = \sec x + C.$
- $\int \sec^2 x \, dx = \tan x + C.$
- $\cos^2\theta = \frac{1+\cos 2\theta}{2}.$
- $\sin^2\theta = \frac{1-\cos 2\theta}{2}.$
- $\sin^2\theta + \cos^2\theta = 1.$
- $\sin 2\theta = 2\sin\theta\cos\theta.$
- $\cos 2\theta = \cos^2\theta - \sin^2\theta.$
- Midpoint Rule: $M(n) = \sum_{k=1}^{n} f\left(\frac{x_{k-1}+x_k}{2}\right)\Delta x.$
- Trapezoid Rule: $T(n) = \left[\frac{1}{2}f(x_0) + \sum_{k=1}^{n-1} f(x_k) + \frac{1}{2}f(x_n)\right]\Delta x.$
- Simpson's Rule: $S(n) = \sum_{k=0}^{n/2-1} [f(x_{2k}) + 4f(x_{2k+1}) + f(x_{2k+2})]\frac{\Delta x}{3}.$

1. (15 points) Circle TRUE if the statement is true or FALSE if it is not, and justify your choice briefly.

   (a) TRUE or FALSE: The partial fractions decomposition of $\dfrac{2x}{x^2+1}$ is $\dfrac{1}{x-1}+\dfrac{1}{x+1}$.
   Explanation:

   (b) TRUE or FALSE: $\displaystyle\int \cos^2 x\,dx = \left(\int \cos x\,dx\right)\left(\int \cos x\,dx\right)$.
   Explanation:

   (c) TRUE or FALSE: The function $y = \cos 2t + \sin 2t$ is a solution of the differential equaiton $y'' + 4y = 0$.
   Explanation:

   (d) TRUE or FALSE: The differential equation

   $$x^x y''(x) + (\ln x \cdot e^{e^x})y'(x) = \frac{y(x)}{x} + \cos x \sin e^x$$

   is linear.
   Explanation:

   (e) TRUE or FALSE: If $a_k > 0$ and $\displaystyle\sum_{k=1}^{\infty} a_k$ converges, then $\displaystyle\sum_{k=1}^{\infty}(-621^{2019})a_k$ diverges.
   Explanation:

2. (20 points) Evaluate the following integrals and sums.

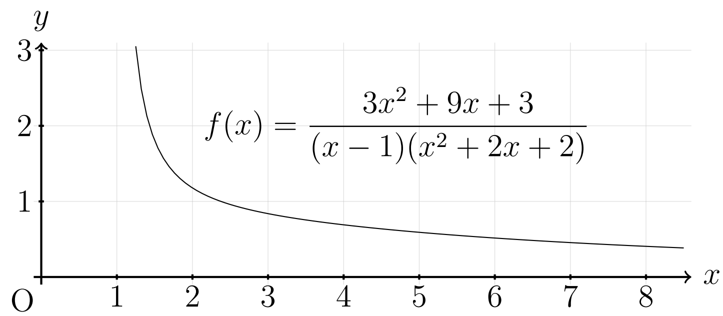
   (a) $\displaystyle\int \ln x \, dx.$

   (b) $\displaystyle\int_0^{\pi/2} \sin^7 x \cos^3 x \, dx.$

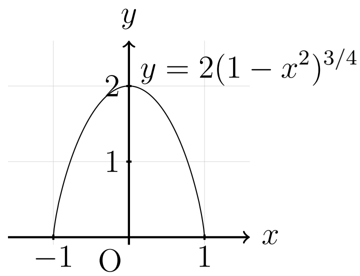
(c) $\displaystyle\sum_{k=2}^{\infty} \frac{28}{9}\left(-\frac{4}{3}\right)^{-k}$.

(d) $\displaystyle\frac{5}{4} + \frac{10}{12} + \frac{20}{36} + \frac{40}{108} + \cdots$.

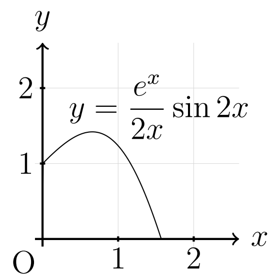
3. (10 points) Compute the area of region R bounded by $f(x) = \dfrac{3x^2+9x+3}{(x-1)(x^2+2x+2)}$, x-axis, $x = 2$, $x = 8$. Indicate (by shading) the region R in the graph.



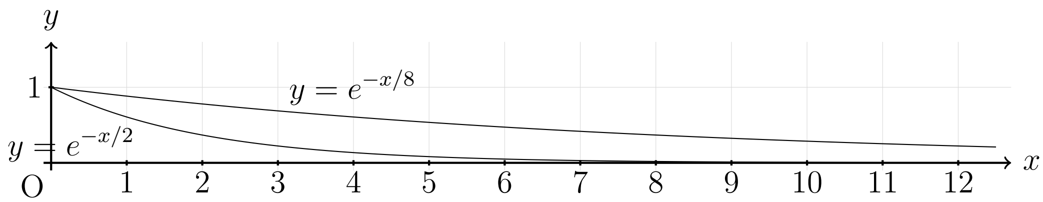
4. (10 points) Let R be the region bounded by $y = 2(1 - x^2)^{3/4}$ and x-axis. Indicate (by shading) the region R in the graph. Determine the volume of the solid of revolution obtained when R is revolved about the x-axis.



5. (10 points) Let R be the region bounded by $y = \dfrac{e^x}{2x}\sin 2x$ and x-axis on $[0, \pi/2]$. Indicate (by shading) the region R in the graph. Determine the volume of the solid of revolution obtained when R is revolved about the y-axis.



6. (10 points) Let R be the region bounded by the graph of $y = e^{-x/2}$ and $y = e^{-x/8}$, for $x \geq 0$. Indicate (by shading) the region R in the graph. What is the volume of the solid generated when R is revolved about the x-axis?



7. (10 points) Write $1.\overline{45} = 1.45454545\ldots$ as a geometric series and express its value as fraciton.

8. (15 points) The velocity of an object moving through a fluid can be modeled by the *drag equation*
$$\frac{dv}{dt} = -\frac{1}{16}v^2.$$
   (a) Find the general solution to this equation.
   (b) An object moving through the water has an initial velocity of 16 m/sec, i.e., $v(0) = 16$. What will the velocity be after 15 seconds?

9. (Bonus, 15 points) Approximate the arc length of $f(x) = \frac{2}{3}(x-1)^{3/2}$ on the interval $[1, 5]$ using Trapezoid Rule with $n = 4$ subintervals. What is the absolute error of the approximation?