Full Name: ______________________________ **W-Number:** ____________________

MATH 2205: Calculus II – Midterm Exam 1

Summer 2019 – Friday, June 07, 2019

Instructions:

- Show all your work and use the space provided on the exam. Correct mathematical notation is required and all partial credit is at discretion of the grader.
- Write neatly and make sure your work is organized.
- Make certain that you have written your Full Name and W-Number in the spaces provided at the top of the exam. Failure to do so may result in a loss of points.
- No aids beyond a scientific, non-graphing calculator are allowed. This means no notes, no cell phones, etc., are permitted during the exam.
- Present your Photo I.D. when turning in your exam.
- The exam has 9 pages. Please check to see that your copy has all the pages.

For Instructor Use Only

Question	1	2	3	4	5	6	7	8	Total
Points	15	20	10	10	10	10	10	15	100
Mark									

Formulae you may find useful:

- $\displaystyle\sum_{k=1}^{n} c = cn, \quad \sum_{k=1}^{n} k = \frac{n(n+1)}{2}, \quad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}.$
- $\displaystyle\int x^p\,dx = \frac{x^{p+1}}{p+1} + C$, where $p \neq -1$.
- $\displaystyle\int x^{-1}\,dx = \ln|x| + C.$
- $\displaystyle\int e^x\,dx = e^x + C.$
- $\displaystyle\int \sin x\,dx = -\cos x + C.$
- $\displaystyle\int \cos x\,dx = \sin x + C.$
- $\displaystyle\int \frac{1}{1+x^2}\,dx = \arctan x + C.$
- $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C.$
- $\displaystyle\int \sec x \tan x\,dx = \sec x + C.$
- $\displaystyle\int \sec^2 x\,dx = \tan x + C.$
- $\displaystyle\cos^2\theta = \frac{1+\cos 2\theta}{2}.$
- $\displaystyle\sin^2\theta = \frac{1-\cos 2\theta}{2}.$
- $\sin^2\theta + \cos^2\theta = 1.$
- $\sin 2\theta = 2\sin\theta\cos\theta.$
- $\cos 2\theta = \cos^2\theta - \sin^2\theta.$

1. (15 points) Circle TRUE if the statement is true or FALSE if it is not, and justify your choice briefly.

   (a) TRUE or FALSE: Suppose that f is defined on the interval $[a, b]$ which is partitioned to n subintervals of equal length. The right Riemann sum for f on $[a, b]$ is always greater than the left Riemann sum for f on $[a, b]$.

   (b) TRUE or FALSE: The piecewise function $f(x) = \begin{cases} x & \text{if } 0 \le x \le 1 \\ -3x+6 & \text{if } 1 < x \le 2 \end{cases}$ is *not* integrable on $[0, 2]$.

   (c) TRUE or FALSE: $\displaystyle\int_0^1 e^{x+\ln 2}\, dx = 2\int_0^1 e^x\, dx.$

   (d) TRUE or FALSE: If the function f is always nonnegative on the interval $[a, b]$, then the area and the net area between the curve and the x-axis from a to b are equal.

   (e) TRUE or FALSE: $\displaystyle\sum_{k=-2}^{1000000} (k+2)(k+1)k = \sum_{k=1}^{1000000} (k+2)(k+1)k.$

2. (20 points) Evaluate the following integrals.

(a) $\displaystyle\sum_{k=1}^{20}(3k+2)^2$.

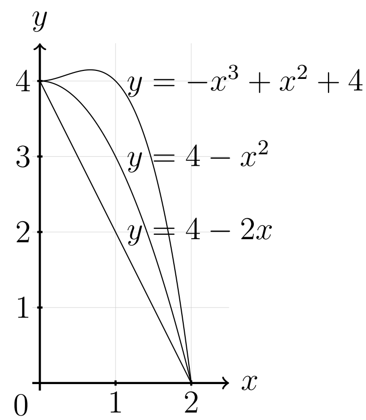
(b) $\displaystyle\sum_{k=11}^{50} 3(2k+1)$.

(c) $\displaystyle\int_{-1}^{3} x\sin(x^2+2)\,dx.$

(d) $\displaystyle\int_{-\pi/2}^{\pi/2} x(e^x+e^{-x})+2\cos^2(x)\,dx.$

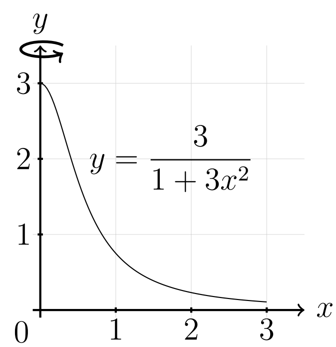
3. (10 points) Use right Riemann Sum to approximate the net area of the region bounded by the graph of $f(x) = (x+4)(x-4)x$ and x-axis on $[0, 20]$ for $n = 10$.

4. (10 points) Here is a picture containing the graphs of three functions. Which is larger,



the area between the curves $y = -x^3 + x^2 + 4$ and $y = 4 - x^2$, or the area between the curves $y = 4 - x^2$ and $y = 4 - 2x$?

5. (10 points) Let R be the region bounded by $y = \dfrac{3}{1+3x^2}$, $x = 1$, x-axis and y-axis. Indicate (by shading) the region R in the graph below. Find the volume of the solid generated when R is revolved about the y-axis.



6. (10 points) Find the arc length of the curve $y = \frac{1}{2}(e^x + e^{-x})$ on $[-\ln 3, \ln 3]$ by integrating with respect to x.

7. (10 points) The graph of $f(x) = 5\sqrt{x}$ on the interval $[1, 3]$ is revolved about the x-axis. What is the area of the surface generated?

8. (15 points) *Physical Applications.*

   (a) Find the mass of the thin bar with the given density function $\rho(x) = \dfrac{4}{1+x^2}$ for $0 \le x \le 1$.

   (b) A spring, which obeys Hooke's law, on a horizontal surface can be stretched and held 0.4 m from its equilibrium position with a force of 40 N. Is the work done in stretching the spring 0.25 m from its equilibrium position equal to the work done in stretching the spring 0.35 m if it has already been stretched 0.1 from its equilibrium position?