MATH 3341 — Spring 2020

Lab 12: Romberg Integration

1. Download the file `Math.3341.Lab.12.zip`, un-zip it.

2. The algorithm for Romberg integration is given below (see Algorithm 1). Please implement the algorithm in MATLAB in the provided function file `lab_12_romberg.m`.

Algorithm 1: Romberg integration: approximates $I = \int_a^b f(x)\,dx$ using n intervals.

Function romberg(f, a, b, n)**:**
  Input : f is the integrand, a is the lower bound, b is the upper bound, n is the number of subintervals.
  Output: The integral of $f(x)$ over the interval $[a, b]$ using $1, 2, 3 \ldots, n$ subintervals.
  $h \leftarrow b - a$;
  $R_{1,1} \leftarrow [f(a) + f(b)] \cdot h/2$;
  for $k \leftarrow 2$ **to** n **do**
    $$R_{k,1} \leftarrow \frac{1}{2}\left[R_{k-1,1} + h\sum_{j=1}^{2^{k-2}} f(a + (2j-1)\cdot h/2)\right];$$
    for $j \leftarrow 2$ **to** k **do**
      $$R_{k,j} \leftarrow R_{k,j-1} + \frac{R_{k,j-1} - R_{k-1,j-1}}{4^{j-1} - 1};$$
    end
    $h \leftarrow h/2$;
  end
  return $[R_{1,1}, R_{2,2}, R_{3,3}, \ldots, R_{n,n}]$;
end

3. Run the script file `lab_12_script.m` to verify your function is working.

4. Uncomment line 18 through line 52 in the script file `lab_12_script.m`, and add more test functions to `lab_12_script.m`:

   (a) $\displaystyle\int_0^{\pi} x^3 \sin x\,dx.$

   (b) $\displaystyle\int_1^{5} x^3 (\ln x)^2\,dx.$

   (c) $\displaystyle\int_{e^e}^{e^4} \ln\ln\ln x\,dx.$

5. Add plots for Romberg integration error of test functions $g(x)$, $h(x)$, $p(x)$ against n, which are also indicated in the comments (around line 65 through line 69).

6. Run `diary('lab_12_output.txt')`, run the script file `lab_12_script.m`, then call `diary off` to save the output to the specified text file.

7. Upload `lab_12_output.txt`, `lab_12_figure.pdf`, `lab_12_script.m`, and `lab_12_romberg.m` to Overleaf, recompile and submit the .pdf report to WyoCourses.