

MATH 3341 — Spring 2020

Lab 06: LU Decomposition

Download `Math.3341.Lab.06.zip`, unzip it by following the Windows Instructions on WyoCourses. Change the current working directory of MATLAB to the unzipped folder, and type `edit lab_06_script` in the Command Window.

1 SOLVE A SYSTEM WITH LU DECOMPOSITION

(a) Define matrix \mathbf{A} and vector \mathbf{b} as (1.1).

$$\underbrace{\begin{bmatrix} 7 & -26 & 45 & -47 \\ 1 & 2 & 3 & 4 \\ 2 & -11 & -12 & -13 \\ 4 & -17 & 30 & 35 \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_{\mathbf{x}} = \underbrace{\begin{bmatrix} -98 \\ 30 \\ -108 \\ 200 \end{bmatrix}}_{\mathbf{b}} \quad (1.1)$$

(b) Calculate the LU decomposition \mathbf{L} , \mathbf{U} of the matrix \mathbf{A} .

(c) Solve the following system (1.2) and store the solution to \mathbf{z} .

$$\mathbf{Lz} = \mathbf{b}. \quad (1.2)$$

(d) Then solve the following system (1.3) and store the solution to \mathbf{x} .

$$\mathbf{Ux} = \mathbf{z}. \quad (1.3)$$

(e) Check your solution by calculating the norm of the residual $\|\mathbf{Ax} - \mathbf{b}\|_2$ and store the result to `res`.

2 VARYING THE VECTOR \mathbf{b}

Suppose we want to solve the system for each integer value of m in between $m = 0$ and $m = 20$. This time use the LU decomposition of the system matrix; perform the decomposition only once and use the lower and upper triangular factors repeatedly to find each successive solution. Then generate a table (Table 1) and a plot (Figure 1) of the solution versus the integer m .

$$\begin{cases} 3x + y + z = m \\ x - 5y + 2z = 5 \\ 2x + y + 5z = 10 \end{cases} \quad (2.1)$$

To do this you'll follow the steps below:

(a) Define coefficient matrix \mathbf{A} given in (2.1), and get the LU decomposition \mathbf{L} , \mathbf{U} of the matrix \mathbf{A} .

(b) Define a vector `m` which ranges from 0 to 20 with step size 1.

- (c) Then create a for-loop, of which the loop iterator i starts from 1 to `length(m)`. In the body of the loop, define a column vector \mathbf{b} as the right-hand side of (2.1), where m should be the i th component of \mathbf{m} . Then repeat (c) and (d) in Part 1. Store the solution \mathbf{x} to the i th row of \mathbf{X} .
- (d) Format the output of \mathbf{m} and \mathbf{X} to a file called `solution.tex` as you did in Part 3 of Lab 05:
- (i) Use `fprintf` to print out the setup for the `table` and `tabular` environments. The first column of the table is centered while the rest three columns are right-justified in L^AT_EX.
 - (ii) Between `\toprule` and `\midrule`, use `fprintf` to print out the heading of the table. The column widths are 4, 11, 11, 11, respectively.
 - (iii) Between `\midrule` and `\bottomrule`, use a for-loop to print each row of the table. Note that the i th row of the table consists of the i th component of \mathbf{m} and the i th row of matrix \mathbf{X} . The column widths are 2, 9, 9, 9, respectively. For floating point numbers, output 6 digits after the decimal point.
 - (iv) Call `type('solution.tex')` to print the content of `solution.tex`.
- (e) Plot the solution versus m using a for-loop as you did in Part 4 of Lab 05:
- (i) Get the size of \mathbf{X} and assign it to `X_size`. Define a cell array `styles`, of which the entries are dashed line with hexagram, dotted line with pentagram, solid line with diamond.
 - (ii) The use a for-loop to plot each column of \mathbf{X} versus \mathbf{m} in the same figure window with the above styles.
 - (iii) Add labels, title, grid, legend as shown in Figure 1.
 - (iv) Save the plot to a file named `lab_06_plot.pdf`.

Type `diary('lab_06_output.txt')` in the Command Window, run the script file `lab_06_script.m`, and type `diary off` in the Command Window. Upload `lab_06_output.txt`, `lab_06_script.m`, `solution.tex`, and `lab_06_plot.pdf` to the folder `src` on Overleaf.

On Overleaf, open `body.tex` under the folder `LaTeX`. In the last section of the report, you will reproduce Section 3 using L^AT_EX. You may find the following helpful:

- You may use environments such as `equation`, `cases`, `figure`, and `table`.
- You may use `\includegraphics[width=amount unit]{/path/to/figure.pdf}` to specify the width of a figure. In our case, the width of the figure is `0.85\textwidth`.
- You may use `\ref{labelName}` to refer to figures, tables; use `\eqref{labelName}` to refer to equations.
- For special symbols, you may look them up in [L^AT_EX.Mathematics.Symbols.pdf](#).
- You may use `\input{/path/to/solution.tex}` to include the table you got from MATLAB.

Recompile and submit the PDF file generated by Overleaf to WyoCourses.

3 BASICS OF L^AT_EX

3.1 LU DECOMPOSITION

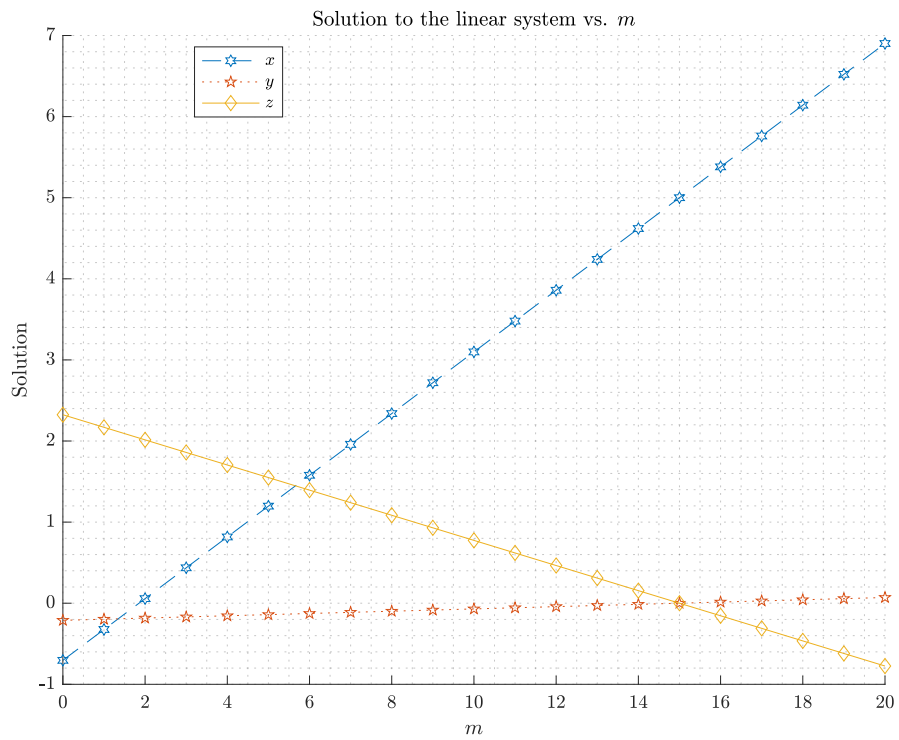
Given the linear system (3.1)

$$\begin{cases} 3x + y + z = m \\ x - 5y + 2z = 5 \\ 2x + y + 5z = 10 \end{cases} \quad (3.1)$$

where $m = 0, 1, 2, \dots, 20$. Using LU Decomposition we can obtain the solution to the linear system (3.1) for corresponding m (see Table 1 and Figure 1).

Table 1: Solution to the linear system

m	x	y	z
0	-0.704225	-0.211268	2.323944
1	-0.323944	-0.197183	2.169014
2	0.056338	-0.183099	2.014085
3	0.436620	-0.169014	1.859155
4	0.816901	-0.154930	1.704225
5	1.197183	-0.140845	1.549296
6	1.577465	-0.126761	1.394366
7	1.957746	-0.112676	1.239437
8	2.338028	-0.098592	1.084507
9	2.718310	-0.084507	0.929577
10	3.098592	-0.070423	0.774648
11	3.478873	-0.056338	0.619718
12	3.859155	-0.042254	0.464789
13	4.239437	-0.028169	0.309859
14	4.619718	-0.014085	0.154930
15	5.000000	-0.000000	0.000000
16	5.380282	0.014085	-0.154930
17	5.760563	0.028169	-0.309859
18	6.140845	0.042254	-0.464789
19	6.521127	0.056338	-0.619718
20	6.901408	0.070423	-0.774648

Figure 1: Solution to the linear system vs. m