MATH 3340 - Scientific Computing Final Project

Due: Friday, 05/15/2020, 05:00 PM

Submit online; make sure to sign the clause. All your work, including the signed page, should be together as one PDF file.

Instruction

1. Go to `https://www.overleaf.com` and sign in (required).
2. Click *Menu* (up left corner), then *Copy Project*.
3. Go to `LaTeX/meta.tex` (the file `meta.tex` under the folder `LaTeX`) to change the section and your name, e.g.,
   - change author to `\author{Albert Einstein \& Carl F. Gauss}`
4. You need to write function/script files, store results to output files. Here are suggested names for function files, script files, output files, and figure files:

Problem	Function File	Script File	Output File
1	`simpson.m` & `gaussSimpson.m`	`final_p1.m`	`final_p1.txt`
2	`steepestDescent.m`	`final_p2.m`	`final_p2.txt`

   Once finished, you need to upload these files to the folder `src` on Overleaf. If you have different filenames, please update the filenames in `\lstinputlisting{../src/your_script_name.m}` accordingly. You can code in the provided files in final.zip, and use the MATLAB script `save_results.m` to generate the output files and store the graphs to `.pdf` files automatically (the script filenames should be exactly same as listed above).
5. Recompile, and download the generated `.pdf` file.
6. **Important**: Enter your name and the date in the boxes above *before* you submit it on WyoCourses.

Problem 1. Write a MATLAB code that uses the parameterization method to calculate the value of the area integral

$$I = \int_{-1}^{1} \int_{x^2}^{4-x^2} [x^2 y + e^{-(x^2+y^2)}]\, dydx.$$

Your code should use Gauss quadrature in the x-direction (the outer integral) and Simpson's rule in the y-direction (the inner integral). For the Gauss integral, the code should allow you to choose up to six integration points (i.e., you should have the option $N = 1, N = 2, \ldots, N = 6$). Similarly, for the inner integral, you should be able to choose an arbitrary number of panels, K; the number of patches will then be $n = 2K$ with the evaluation points numbered $y_0, y_1, y_2, \ldots, y_n$ as in the notes. Run the code with $N = 2$ and $K = 2$, then with $N = 6$ and $K = 4$. Print the results you obtained for these two cases. Do not round off your results to four decimal places, the MATLAB default; instead, print all available decimals. Indicate clearly the values of N and K used for each calculation and submit all your code used for this problem. While this is your choice, it is probably best to organize it as a script calling the pertaining function (or functions), as usual.

Problem 2. Solve the linear system of equations:

$$\begin{cases} 3x + y - z = 4 \\ x + 5y + 2z = -1 \\ -x + 2y + 5z = 1 \end{cases}$$

using the steepest descent method starting from the usual initial guess $\mathbf{x}^0 = [0, 0, 0]^T$. For each iteration, print the iteration number k, the current approximation $\mathbf{x}^k$, and the norm of the residual at this current location. Start with $k = 0$ and produce a table with this information. Convergence should be declared when the norm of the residual is below 10^{-7}. Submit all pieces of code that you used to solve the problem, together with the aforementioned results.