

Formulae you may find useful:

- $\sum_{k=1}^n c = cn$, $\sum_{k=1}^n k = \frac{n(n+1)}{2}$, $\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$, $\sum_{k=1}^n k^3 = \frac{n^2(n+1)^2}{4}$.
- $\int x^p dx = \frac{x^{p+1}}{p+1} + C$, where $p \neq -1$.
- $\int x^{-1} dx = \ln|x| + C$.
- $\int e^x dx = e^x + C$.
- $\int \sin x dx = -\cos x + C$.
- $\int \cos x dx = \sin x + C$.
- $\int \frac{1}{1+x^2} dx = \arctan x + C$.
- $\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C$.
- $\int \sec x \tan x dx = \sec x + C$.
- $\int \sec^2 x dx = \tan x + C$.
- $\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$.
- $\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$.
- $\sin^2 \theta + \cos^2 \theta = 1$.
- $\sin 2\theta = 2 \sin \theta \cos \theta$.
- $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$.

1. (15 points) Circle TRUE if the statement is true or FALSE if it is not, and justify your choice briefly.

(a) TRUE or FALSE: If the function f is always negative on the interval $[a, b]$, then the area and the net area between the curve and the x -axis from a to b are equal.

(b) TRUE or FALSE: If the average value of a function on $[a, b]$ is positive, then that function cannot be negative on that interval.

(c) TRUE or FALSE: We could use substitution rule to evaluate $\int_0^1 e^x \cos e^x dx$.

(d) TRUE or FALSE: Given a region in the first quadrant, it is possible that the volume of the solid obtained by revolving about the x -axis is equal to the volume of the solid obtained by revolving about the y -axis.

(e) TRUE or FALSE: The Fundamental Theorem of Calculus uses the derivatives of the function f to evaluate the definite integral $\int_a^b f(x) dx$.

2. (20 points) Evaluate the following sums and integrals.

(a) $\sum_{k=1}^{10} (2k + 3)k.$

(b) $\sum_{k=-2}^5 (k + 1)(k + 2)k.$

(c) $\int_{-3}^3 5x^4 + x^2 \sin^3(x) dx.$

(d) $\int_0^\pi e^x \sin^2(2e^x) dx.$

3. (10 points) Use right Riemann Sum to approximate the net area of the region bounded by the graph of $f(x) = 3x(x^2 + 2) + 5$ and x -axis on $[0, 5]$ for $n = 5$.

4. (10 points) Let \mathcal{R} be the region in the first quadrant between the curves $y = x$ and $y = x^n$, where $n > 1$. Find the value of n so that the area of \mathcal{R} is equal to $\frac{1}{10}$.

5. (10 points) Let R be the region bounded by the following curves: $y = x^2$, $y = 2 - x$, and $x = 0$ in the first quadrant; revolved about the y -axis. Find the volume of the solid.

6. (10 points) Find the arc length of the curve $y = \frac{1}{2}(e^x + e^{-x})$ on $[-\ln 2, \ln 2]$ by integrating with respect to x .

7. (10 points) *Surface area of an ellipsoid* - If the top half of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is revolved about the x -axis, the result is an *ellipsoid* whose axis along the x -axis has length $2a$, whose axis along the y -axis has length $2b$, and whose axis perpendicular to the xy -plane has length $2b$. We assume that $0 < b < a$. Find the surface area of the ellipsoid for $a = 4$, $b = 2$.

8. (15 points) *Nonlinear Springs* - Since Hooke's law is only an approximation to the spring force when the size of the displacement is small, we can increase the accuracy of our model by increasing its complexity. Let's think about a spring whose restoring force is given by

$$F_s(x) = 16x - 0.1x^3$$

for $-6 \leq x \leq 6$, where x is the displacement of the spring from equilibrium.

- (a) How much work is done in stretching the spring from its equilibrium position ($x = 0$) to $x = 4.5$?
- (b) Compare the result for the previous part with the result that you would get if we used Hooke's law, i.e., $F_h(x) = 16x$.

Topics for Midterm Exam 1:

- Sigma Notation;
- Left/midpoint/right Riemann Sum;
- Definite Integral;
- Fundamental Theorem of Calculus;
- Integrating with Even and Odd Functions;
- Mean Value Theorem for Integrals;
- Substitution Rule for Indefinite/Definite Integrals;
- Area of the Region Between Curves (Integrating with respect to x or y);
- Volume of the Solid by Slicing (Disk/Washer Method) and by Shell Method;
- Length of Curves;
- Surface Area;
- Physical Applications: Mass of a One-Dimensional Object, Work (Hooke's law).