Full Name: ______________________________ **W-Number:** ____________________

MATH 2205: Calculus II – Final Exam

Summer 2019 – Friday, July 05, 2019

Instructions:

- Show all your work and use the space provided on the exam. Correct mathematical notation is required and all partial credit is at discretion of the grader.
- Write neatly and make sure your work is organized.
- Make certain that you have written your Full Name and W-Number in the spaces provided at the top of the exam. Failure to do so may result in a loss of points.
- No aids beyond a scientific, non-graphing calculator are allowed. This means no notes, no cell phones, etc., are permitted during the exam.
- Present your Photo I.D. when turning in your exam.
- The exam has 10 pages. Please check to see that your copy has all the pages.

For Instructor Use Only

Question	1	2	3	4	5	6	7	8	9	Total
Points	15	20	10	10	10	10	10	15	10	100
Mark										

Formulae you may find useful:

- $\displaystyle\sum_{k=1}^{n} c = cn, \quad \sum_{k=1}^{n} k = \frac{n(n+1)}{2}, \quad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}.$
- $\displaystyle\int x^p\,dx = \frac{x^{p+1}}{p+1} + C$, where $p \neq -1$.
- $\displaystyle\int x^{-1}\,dx = \ln|x| + C.$
- $\displaystyle\int e^x\,dx = e^x + C.$
- $\displaystyle\int \sin x\,dx = -\cos x + C.$
- $\displaystyle\int \cos x\,dx = \sin x + C.$
- $\displaystyle\int \frac{1}{1+x^2}\,dx = \arctan x + C.$
- $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C.$
- $\displaystyle\int \sec x \tan x\,dx = \sec x + C.$
- $\displaystyle\int \sec^2 x\,dx = \tan x + C.$
- $\displaystyle\cos^2\theta = \frac{1+\cos 2\theta}{2}.$
- $\displaystyle\sin^2\theta = \frac{1-\cos 2\theta}{2}.$
- $\sin^2\theta + \cos^2\theta = 1.$
- $\sin 2\theta = 2\sin\theta\cos\theta.$
- $\cos 2\theta = \cos^2\theta - \sin^2\theta.$
- Midpoint Rule: $\displaystyle M(n) = \sum_{k=1}^{n} f\left(\frac{x_{k-1}+x_k}{2}\right)\Delta x.$
- Trapezoid Rule: $\displaystyle T(n) = \left[\frac{1}{2}f(x_0) + \sum_{k=1}^{n-1} f(x_k) + \frac{1}{2}f(x_n)\right]\Delta x.$
- Simpson's Rule: $\displaystyle S(n) = \sum_{k=0}^{n/2-1} \left[f(x_{2k}) + 4f(x_{2k+1}) + f(x_{2k+2})\right]\frac{\Delta x}{3}.$

1. (15 points) Circle TRUE if the statement is true or FALSE if it is not, and justify your choice briefly.

   (a) TRUE or FALSE: The partial fractions decomposition of $\dfrac{1}{(4k+1)(4k+3)}$ is $\dfrac{1}{4k+1} - \dfrac{1}{4k+3}$.
   Explanation:

   (b) TRUE or FALSE: The interval of convergence of the series $\displaystyle\sum_{k=1}^{\infty} c_k(x-a)^k$ must contain the center $x = a$.
   Explanation:

   (c) TRUE or FALSE: The Fundamental Theorem of Calculus uses the derivative of the function f to evaluate the definite integral $\displaystyle\int_a^b f(x)\,dx$.
   Explanation:

   (d) TRUE or FALSE: If $\displaystyle\sum_{k=0}^{\infty}(\cos k \cdot a_k)$ diverges, then $\displaystyle\sum_{k=0}^{\infty}|a_k|$ must diverge.
   Explanation:

   (e) TRUE or FALSE: $\dfrac{[2(k+1)]!}{(2k)!} = (2k+1)(2k+2)$.
   Explanation:

2. (20 points) Evaluate the following sums.

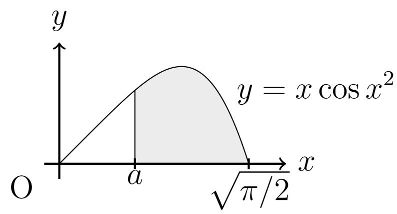
(a) $\displaystyle\sum_{k=11}^{50} (k+3)(k-3).$

(b) $\displaystyle\sum_{k=0}^{\infty} \frac{3}{(3k+1)(3k+4)}.$

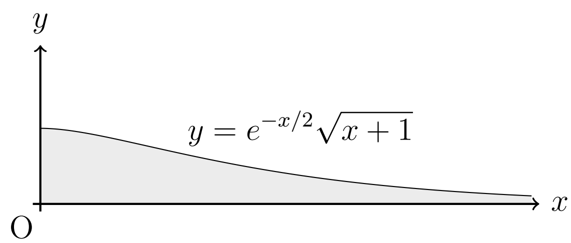
(c) $\dfrac{1}{3} + \dfrac{2}{9} + \dfrac{4}{27} + \dfrac{8}{81} + \cdots.$

(d) $\displaystyle\sum_{k=0}^{\infty} \left(e^{-2k-1} + \frac{1}{\pi^{3k+1}} \right).$

3. (10 points) Determine the value of the positive parameter a so that the area of the shaded region in the picture is equal to $\frac{1}{2}$.



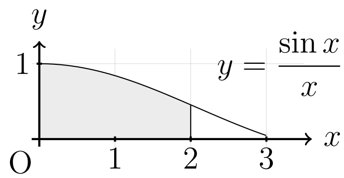
4. (10 points) Consider the infinitely long shaded region R indicated in the picture. Determine the volume of the solid of the revolution obtained when R is revolved about the x-axis.



5. (10 points) Find the interval of convergence and radius of convergence for the power series given by

$$\sum_{k=1}^{\infty} \frac{(-1)^k (x-1)^k}{k^{4/5}}.$$

6. (10 points) Use MacLaurin Series to approximate the net area between the function $y = \dfrac{\sin x}{x}$ and the x-axis from $x = 0$ to $x = 2$ with an error no greater than $10^{-4} = 0.0001$. Be sure to justify that your error satisfies the given bound.



7. (10 points) Find MacLaurin Series for $\ln(1+2x)$ and give its interval of convergence.

8. (15 points) Determine whether the following series converge.

(a) $\displaystyle\sum_{k=1}^{\infty}\left(\frac{k^4+10k}{2k^4+50}\right)^k.$

(b) $\displaystyle\sum_{k=1}^{\infty}\frac{k^k}{k!}.$

9. (Bonus, 10 points) Compute the area of region R bounded by $y = \dfrac{x^2+3x+1}{(x-1)(x^2+2x+2)}$, x-axis, $x = 2$, $x = 8$.